

Twelve Real Dirac Matrices

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Abstract

The Dirac matrices can be interpreted as a set of basis for geometric algebra in a Minkowski metric space. The imaginary factors in Dirac's γ^2 matrix have always annoyed me. In this note, I present twelve alternatives to the Dirac matrices, using simple real numbers, for a metric $(+,+,+,-)$.

Minkowski Space Time Algebra

I wrote a simple program to scan through 4x4 matrices identifying a set of matrices suitable as a basis for Minkowski space time algebra using a metric signature of $(+,+,+,-)$.

Having found one basis, I found another 11 be simple permutations among the matrix elements. The program <http://www.kurtnalty.com/Minkowski.c> and output file <http://www.kurtnalty.com/Minkowski.txt> lists the matrix elements, and explicitly the twelve Minkowski 4x4 real matrices.

These matrices, their generators inside the Minkowski program, and their nicknames are shown below. These matrices make a complete basis set for 4x4 matrices.

35931560 (P1)
 [1 0 0 0]
 [0 1 0 0]
 [0 0 1 0]
 [0 0 0 1]

16563328 (+a) 35813460 (+b) 22075240 (+c) 23108944 (+D)
 [0 1 0 0] [-1 0 0 0] [0 0 0 1] [0 0 1 0]
 [1 0 0 0] [0 1 0 0] [0 0 1 0] [0 0 0 -1]
 [0 0 0 -1] [0 0 -1 0] [0 1 0 0] [-1 0 0 0]
 [0 0 -1 0] [0 0 0 1] [1 0 0 0] [0 1 0 0]

35812976 (+d) 26483560 (+e) 19933420 (+f) 22073728 (+E)
 [1 0 0 0] [0 1 0 0] [0 0 1 0] [0 0 0 -1]
 [0 -1 0 0] [1 0 0 0] [0 0 0 -1] [0 0 -1 0]
 [0 0 -1 0] [0 0 0 1] [1 0 0 0] [0 1 0 0]
 [0 0 0 1] [0 0 1 0] [0 -1 0 0] [1 0 0 0]

23126440 (+g) 21012304 (+h) 35931072 (+i) 26129260 (+F)
 [0 0 1 0] [0 0 0 -1] [-1 0 0 0] [0 -1 0 0]
 [0 0 0 1] [0 0 1 0] [0 -1 0 0] [1 0 0 0]
 [1 0 0 0] [0 1 0 0] [0 0 1 0] [0 0 0 -1]
 [0 1 0 0] [-1 0 0 0] [0 0 0 1] [0 0 1 0]

22035820 (+C) 23122048 (+B) 26129104 (+A)
 [0 0 0 -1] [0 0 -1 0] [0 1 0 0]
 [0 0 1 0] [0 0 0 -1] [-1 0 0 0]
 [0 -1 0 0] [1 0 0 0] [0 0 0 -1]
 [1 0 0 0] [0 1 0 0] [0 0 1 0]

The nine lower case matrices square to unity. The six upper case matrices square to negative one. We can arrange the nine unity roots as shown above in a 3x3 grid, such that the triple products across and down result in the imaginary roots shown right and below.

In nickname form, this multiplication format is shown below.

(+a)(+b)(+c) | (+D)
 (+d)(+e)(+f) | (+E)
 (+g)(+h)(+i) | (+F)

 (+C)(+B)(+A)

The full multiplication table among these elements is

Products Among the 16

	(P1)	(+a)	(+b)	(+c)	(+d)	(+e)	(+f)	(+g)	(+h)	(+i)	(+A)	(+B)	(+C)	(+D)	(+E)	(+F)
(P1)	(P1)	(+a)	(+b)	(+c)	(+d)	(+e)	(+f)	(+g)	(+h)	(+i)	(+A)	(+B)	(+C)	(+D)	(+E)	(+F)
(+a)	(+a)	(P1)	(+A)	(-B)	(+F)	(-i)	(+h)	(-E)	(+f)	(-e)	(+b)	(-c)	(+D)	(+C)	(-g)	(+d)
(+b)	(+b)	(-A)	(P1)	(+C)	(+i)	(+F)	(-g)	(-f)	(-E)	(+d)	(-a)	(+D)	(+c)	(+B)	(-h)	(+e)
(+c)	(+c)	(+B)	(-C)	(P1)	(-h)	(+g)	(+F)	(+e)	(-d)	(-E)	(+D)	(+a)	(-b)	(+A)	(-i)	(+f)
(+d)	(+d)	(-F)	(+i)	(-h)	(P1)	(+A)	(-B)	(+D)	(-c)	(+b)	(+e)	(-f)	(+E)	(+g)	(+C)	(-a)
(+e)	(+e)	(-i)	(-F)	(+g)	(-A)	(P1)	(+C)	(+c)	(+D)	(-a)	(-d)	(+E)	(+f)	(+h)	(+B)	(-b)
(+f)	(+f)	(+h)	(-g)	(-F)	(+B)	(-C)	(P1)	(-b)	(+a)	(+D)	(+E)	(+d)	(-e)	(+i)	(+A)	(-c)
(+g)	(+g)	(+E)	(-f)	(+e)	(-D)	(+c)	(-b)	(P1)	(+A)	(-B)	(+h)	(-i)	(+F)	(-d)	(+a)	(+C)
(+h)	(+h)	(+f)	(+E)	(-d)	(-c)	(-D)	(+a)	(-A)	(P1)	(+C)	(-g)	(+F)	(+i)	(-e)	(+b)	(+B)
(+i)	(+i)	(-e)	(+d)	(+E)	(+b)	(-a)	(-D)	(+B)	(-C)	(P1)	(+F)	(+g)	(-h)	(-f)	(+c)	(+A)
(+A)	(+A)	(-b)	(+a)	(+D)	(-e)	(+d)	(+E)	(-h)	(+g)	(+F)	(M1)	(+C)	(-B)	(-c)	(-f)	(-i)
(+B)	(+B)	(+c)	(+D)	(-a)	(+f)	(+E)	(-d)	(+i)	(+F)	(-g)	(-C)	(M1)	(+A)	(-b)	(-e)	(-h)
(+C)	(+C)	(+D)	(-c)	(+b)	(+E)	(-f)	(+e)	(+F)	(-i)	(+h)	(+B)	(-A)	(M1)	(-a)	(-d)	(-g)
(+D)	(+D)	(+C)	(+B)	(+A)	(-g)	(-h)	(-i)	(+d)	(+e)	(+f)	(-c)	(-b)	(-a)	(M1)	(-F)	(+E)
(+E)	(+E)	(+g)	(+h)	(+i)	(+C)	(+B)	(+A)	(-a)	(-b)	(-c)	(-f)	(-e)	(-d)	(+F)	(M1)	(-D)
(+F)	(+F)	(-d)	(-e)	(-f)	(+a)	(+b)	(+c)	(+C)	(+B)	(+A)	(-i)	(-h)	(-g)	(-E)	(+D)	(M1)

Using these matrices, we can generate twelve real implementations of Minkowski geometric algebra, with a signature $(+,+,+,-)$ for (x,y,z,t) . The table below lists twelve arrangements of the above matrices which generate Minkowski geometric algebra, where the higher order products, such as tx or xyz are formed by standard matrix multiplication.

x	y	z	t
a	b	c	E
a	b	c	F
d	e	f	D
d	e	f	F
g	h	i	D
g	h	i	E
a	d	g	B
a	d	g	A
b	e	h	C
b	e	h	A
c	f	i	B
c	f	i	C

A code fragment containing the same information is shown below.

```
int M1 [4]={16563328, 35813460, 22075240, 22073728}; // a b c E
int M2 [4]={16563328, 35813460, 22075240, 26129260}; // a b c F

int M3 [4]={35812976, 26483560, 19933420, 23108944}; // d e f D
int M4 [4]={35812976, 26483560, 19933420, 26129260}; // d e f F

int M5 [4]={23126440, 21012304, 35931072, 23108944}; // g h i D
int M6 [4]={23126440, 21012304, 35931072, 22073728}; // g h i E

int M7 [4]={16563328, 35812976, 23126440, 23122048}; // a d g B
int M8 [4]={16563328, 35812976, 23126440, 26129104}; // a d g A

int M9 [4]={35813460, 26483560, 21012304, 22035820}; // b e h C
int M10[4]={35813460, 26483560, 21012304, 26129104}; // b e h A

int M11[4]={22075240, 19933420, 35931072, 23122048}; // c f i B
int M12[4]={22075240, 19933420, 35931072, 22035820}; // c f i C
```

The next twelve pages list the twelve full geometric algebra Minkowsky implementations.

Implementation #1

	(P1)	(+a)	(+b)	(+c)	(+E)	(+A)	(-B)	(+C)	(-g)	(-h)	(-i)	(+D)	(-f)	(+e)	(-d)	(-F)	
(P1)		(P1)	(+a)	(+b)	(+c)	(+E)	(+A)	(-B)	(+C)	(-g)	(-h)	(-i)	(+D)	(-f)	(+e)	(-d)	(-F)
(+a)		(+a)	(P1)	(+A)	(-B)	(-g)	(+b)	(+c)	(+D)	(+E)	(-f)	(+e)	(+C)	(-h)	(-i)	(-F)	(-d)
(+b)		(+b)	(-A)	(P1)	(+C)	(-h)	(-a)	(-D)	(+c)	(+f)	(+E)	(-d)	(+B)	(+g)	(+F)	(-i)	(-e)
(+c)		(+c)	(+B)	(-C)	(P1)	(-i)	(+D)	(-a)	(-b)	(-e)	(+d)	(+E)	(+A)	(-F)	(+g)	(+h)	(-f)
(+E)		(+E)	(+g)	(+h)	(+i)	(M1)	(-f)	(+e)	(-d)	(+a)	(+b)	(+c)	(+F)	(-A)	(+B)	(-C)	(+D)
(+A)		(+A)	(-b)	(+a)	(+D)	(-f)	(M1)	(-C)	(-B)	(+h)	(-g)	(-F)	(-c)	(-E)	(+d)	(+e)	(+i)
(-B)		(-B)	(-c)	(-D)	(+a)	(+e)	(+C)	(M1)	(-A)	(+i)	(+F)	(-g)	(+b)	(-d)	(-E)	(+f)	(-h)
(+C)		(+C)	(+D)	(-c)	(+b)	(-d)	(+B)	(+A)	(M1)	(-F)	(+i)	(-h)	(-a)	(-e)	(-f)	(-E)	(+g)
(-g)		(-g)	(-E)	(+f)	(-e)	(-a)	(-h)	(-i)	(-F)	(P1)	(+A)	(-B)	(+d)	(-b)	(-c)	(-D)	(+C)
(-h)		(-h)	(-f)	(-E)	(+d)	(-b)	(+g)	(+F)	(-i)	(-A)	(P1)	(+C)	(+e)	(+a)	(+D)	(-c)	(+B)
(-i)		(-i)	(+e)	(-d)	(-E)	(-c)	(-F)	(+g)	(+h)	(+B)	(-C)	(P1)	(+f)	(-D)	(+a)	(+b)	(+A)
(+D)		(+D)	(+C)	(+B)	(+A)	(-F)	(-c)	(+b)	(-a)	(-d)	(-e)	(-f)	(M1)	(+i)	(-h)	(+g)	(-E)
(-f)		(-f)	(-h)	(+g)	(+F)	(-A)	(-E)	(+d)	(+e)	(-b)	(+a)	(+D)	(-i)	(P1)	(+C)	(+B)	(-c)
(+e)		(+e)	(-i)	(-F)	(+g)	(+B)	(-d)	(-E)	(+f)	(-c)	(-D)	(+a)	(+h)	(-C)	(P1)	(+A)	(+b)
(-d)		(-d)	(+F)	(-i)	(+h)	(-C)	(-e)	(-f)	(-E)	(+D)	(-c)	(+b)	(-g)	(-B)	(-A)	(P1)	(-a)
(-F)		(-F)	(+d)	(+e)	(+f)	(-D)	(+i)	(-h)	(+g)	(+C)	(+B)	(+A)	(+E)	(+c)	(-b)	(+a)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	1	0	0]
[0	1	0	0]	[-1	0	0	0]
[0	0	1	0]	[0	0	0	1]
[0	0	0	1]	[0	0	-1	0]

x				y				z				t			
[0	1	0	0]	[-1	0	0	0]	[0	0	0	1]	[0	0	0	-1]
[1	0	0	0]	[0	1	0	0]	[0	0	1	0]	[0	0	-1	0]
[0	0	0	-1]	[0	0	-1	0]	[0	1	0	0]	[0	1	0	0]
[0	0	-1	0]	[0	0	0	1]	[1	0	0	0]	[1	0	0	0]

xy				xz				yz				xt				yt				zt			
[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	0	-1	0]	[0	0	0	1]	[1	0	0	0]
[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[0	0	0	-1]	[0	0	-1	0]	[0	1	0	0]
[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]	[-1	0	0	0]	[0	-1	0	0]	[0	0	-1	0]
[0	0	1	0]	[0	-1	0	0]	[1	0	0	0]	[0	-1	0	0]	[1	0	0	0]	[0	0	0	-1]

xyz				xyt				xzt				yzt			
[0	0	1	0]	[0	0	-1	0]	[0	1	0	0]	[-1	0	0	0]
[0	0	0	-1]	[0	0	0	1]	[1	0	0	0]	[0	1	0	0]
[-1	0	0	0]	[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]
[0	1	0	0]	[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]

Implementation #2

	(P1)	(+a)	(+b)	(+c)	(+F)	(+A)	(-B)	(+C)	(+d)	(+e)	(+f)	(+D)	(-i)	(+h)	(-g)	(+E)	
(P1)		(P1)	(+a)	(+b)	(+c)	(+F)	(+A)	(-B)	(+C)	(+d)	(+e)	(+f)	(+D)	(-i)	(+h)	(-g)	(+E)
(+a)		(+a)	(P1)	(+A)	(-B)	(+d)	(+b)	(+c)	(+D)	(+F)	(-i)	(+h)	(+C)	(+e)	(+f)	(+E)	(-g)
(+b)		(+b)	(-A)	(P1)	(+C)	(+e)	(-a)	(-D)	(+c)	(+i)	(+F)	(-g)	(+B)	(-d)	(-E)	(+f)	(-h)
(+c)		(+c)	(+B)	(-C)	(P1)	(+f)	(+D)	(-a)	(-b)	(-h)	(+g)	(+F)	(+A)	(+E)	(-d)	(-e)	(-i)
(+F)		(+F)	(-d)	(-e)	(-f)	(M1)	(-i)	(+h)	(-g)	(+a)	(+b)	(+c)	(-E)	(-A)	(+B)	(-C)	(+D)
(+A)		(+A)	(-b)	(+a)	(+D)	(-i)	(M1)	(-C)	(-B)	(-e)	(+d)	(+E)	(-c)	(-F)	(+g)	(+h)	(-f)
(-B)		(-B)	(-c)	(-D)	(+a)	(+h)	(+C)	(M1)	(-A)	(-f)	(-E)	(+d)	(+b)	(-g)	(-F)	(+i)	(+e)
(+C)		(+C)	(+D)	(-c)	(+b)	(-g)	(+B)	(+A)	(M1)	(+E)	(-f)	(+e)	(-a)	(-h)	(-i)	(-F)	(-d)
(+d)		(+d)	(-F)	(+i)	(-h)	(-a)	(+e)	(+f)	(+E)	(P1)	(+A)	(-B)	(+g)	(-b)	(-c)	(-D)	(+C)
(+e)		(+e)	(-i)	(-F)	(+g)	(-b)	(-d)	(-E)	(+f)	(-A)	(P1)	(+C)	(+h)	(+a)	(+D)	(-c)	(+B)
(+f)		(+f)	(+h)	(-g)	(-F)	(-c)	(+E)	(-d)	(-e)	(+B)	(-C)	(P1)	(+i)	(-D)	(+a)	(+b)	(+A)
(+D)		(+D)	(+C)	(+B)	(+A)	(+E)	(-c)	(+b)	(-a)	(-g)	(-h)	(-i)	(M1)	(-f)	(+e)	(-d)	(-F)
(-i)		(-i)	(+e)	(-d)	(-E)	(-A)	(-F)	(+g)	(+h)	(-b)	(+a)	(+D)	(+f)	(P1)	(+C)	(+B)	(-c)
(+h)		(+h)	(+f)	(+E)	(-d)	(+B)	(-g)	(-F)	(+i)	(-c)	(-D)	(+a)	(-e)	(-C)	(P1)	(+A)	(+b)
(-g)		(-g)	(-E)	(+f)	(-e)	(-C)	(-h)	(-i)	(-F)	(+D)	(-c)	(+b)	(+d)	(-B)	(-A)	(P1)	(-a)
(+E)		(+E)	(+g)	(+h)	(+i)	(-D)	(-f)	(+e)	(-d)	(+C)	(+B)	(+A)	(+F)	(+c)	(-b)	(+a)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	0	-1]
[0	1	0	0]	[0	0	-1	0]
[0	0	1	0]	[0	1	0	0]
[0	0	0	1]	[1	0	0	0]

x				y				z				t			
[0	1	0	0]	[-1	0	0	0]	[0	0	0	1]	[0	-1	0	0]
[1	0	0	0]	[0	1	0	0]	[0	0	1	0]	[1	0	0	0]
[0	0	0	-1]	[0	0	-1	0]	[0	1	0	0]	[0	0	0	-1]
[0	0	-1	0]	[0	0	0	1]	[1	0	0	0]	[0	0	1	0]

xy				xz				yz				xt				yt				zt			
[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]	[1	0	0	0]	[0	1	0	0]	[0	0	1	0]
[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]	[1	0	0	0]	[0	0	0	-1]
[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]	[0	0	-1	0]	[0	0	0	1]	[1	0	0	0]
[0	0	1	0]	[0	-1	0	0]	[1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]

xyz				xyt				xzt				yzt			
[0	0	1	0]	[1	0	0	0]	[0	0	0	-1]	[0	0	-1	0]
[0	0	0	-1]	[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]
[-1	0	0	0]	[0	0	-1	0]	[0	1	0	0]	[-1	0	0	0]
[0	1	0	0]	[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]

Implementation #3

	(P1)	(+d)	(+e)	(+f)	(+D)	(+A)	(-B)	(+C)	(+g)	(+h)	(+i)	(+E)	(-c)	(+b)	(-a)	(+F)
(P1)	(P1)	(+d)	(+e)	(+f)	(+D)	(+A)	(-B)	(+C)	(+g)	(+h)	(+i)	(+E)	(-c)	(+b)	(-a)	(+F)
(+d)	(+d)	(P1)	(+A)	(-B)	(+g)	(+e)	(+f)	(+E)	(+D)	(-c)	(+b)	(+C)	(+h)	(+i)	(+F)	(-a)
(+e)	(+e)	(-A)	(P1)	(+C)	(+h)	(-d)	(-E)	(+f)	(+c)	(+D)	(-a)	(+B)	(-g)	(-F)	(+i)	(-b)
(+f)	(+f)	(+B)	(-C)	(P1)	(+i)	(+E)	(-d)	(-e)	(-b)	(+a)	(+D)	(+A)	(+F)	(-g)	(-h)	(-c)
(+D)	(+D)	(-g)	(-h)	(-i)	(M1)	(-c)	(+b)	(-a)	(+d)	(+e)	(+f)	(-F)	(-A)	(+B)	(-C)	(+E)
(+A)	(+A)	(-e)	(+d)	(+E)	(-c)	(M1)	(-C)	(-B)	(-h)	(+g)	(+F)	(-f)	(-D)	(+a)	(+b)	(-i)
(-B)	(-B)	(-f)	(-E)	(+d)	(+b)	(+C)	(M1)	(-A)	(-i)	(-F)	(+g)	(+e)	(-a)	(-D)	(+c)	(+h)
(+C)	(+C)	(+E)	(-f)	(+e)	(-a)	(+B)	(+A)	(M1)	(+F)	(-i)	(+h)	(-d)	(-b)	(-c)	(-D)	(-g)
(+g)	(+g)	(-D)	(+c)	(-b)	(-d)	(+h)	(+i)	(+F)	(P1)	(+A)	(-B)	(+a)	(-e)	(-f)	(-E)	(+C)
(+h)	(+h)	(-c)	(-D)	(+a)	(-e)	(-g)	(-F)	(+i)	(-A)	(P1)	(+C)	(+b)	(+d)	(+E)	(-f)	(+B)
(+i)	(+i)	(+b)	(-a)	(-D)	(-f)	(+F)	(-g)	(-h)	(+B)	(-C)	(P1)	(+c)	(-E)	(+d)	(+e)	(+A)
(+E)	(+E)	(+C)	(+B)	(+A)	(+F)	(-f)	(+e)	(-d)	(-a)	(-b)	(-c)	(M1)	(-i)	(+h)	(-g)	(-D)
(-c)	(-c)	(+h)	(-g)	(-F)	(-A)	(-D)	(+a)	(+b)	(-e)	(+d)	(+E)	(+i)	(P1)	(+C)	(+B)	(-f)
(+b)	(+b)	(+i)	(+F)	(-g)	(+B)	(-a)	(-D)	(+c)	(-f)	(-E)	(+d)	(-h)	(-C)	(P1)	(+A)	(+e)
(-a)	(-a)	(-F)	(+i)	(-h)	(-C)	(-b)	(-c)	(-D)	(+E)	(-f)	(+e)	(+g)	(-B)	(-A)	(P1)	(-d)
(+F)	(+F)	(+a)	(+b)	(+c)	(-E)	(-i)	(+h)	(-g)	(+C)	(+B)	(+A)	(+D)	(+f)	(-e)	(+d)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	-1	0	0]
[0	1	0	0]	[1	0	0	0]
[0	0	1	0]	[0	0	0	-1]
[0	0	0	1]	[0	0	1	0]

x		y		z		t									
[1	0	0	0]	[0	1	0	0]	[0	0	1	0]	[0	0	1	0]
[0	-1	0	0]	[1	0	0	0]	[0	0	0	-1]	[0	0	0	-1]
[0	0	-1	0]	[0	0	0	1]	[1	0	0	0]	[-1	0	0	0]
[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]	[0	1	0	0]

xy		xz		yz		xt		yt		zt													
[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	0	1	0]	[0	0	0	-1]	[-1	0	0	0]
[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]
[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]	[1	0	0	0]	[0	1	0	0]	[0	0	1	0]
[0	0	1	0]	[0	-1	0	0]	[1	0	0	0]	[0	1	0	0]	[-1	0	0	0]	[0	0	0	1]

xyz		xyt		xzt		yzt									
[0	0	0	-1]	[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]
[0	0	-1	0]	[0	0	-1	0]	[0	1	0	0]	[-1	0	0	0]
[0	1	0	0]	[0	-1	0	0]	[0	0	-1	0]	[0	0	0	1]
[1	0	0	0]	[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]

Implementation #4

	(P1)	(+d)	(+e)	(+f)	(+F)	(+A)	(-B)	(+C)	(-a)	(-b)	(-c)	(+E)	(-i)	(+h)	(-g)	(-D)
(P1)	(P1)	(+d)	(+e)	(+f)	(+F)	(+A)	(-B)	(+C)	(-a)	(-b)	(-c)	(+E)	(-i)	(+h)	(-g)	(-D)
(+d)	(+d)	(P1)	(+A)	(-B)	(-a)	(+e)	(+f)	(+E)	(+F)	(-i)	(+h)	(+C)	(-b)	(-c)	(-D)	(-g)
(+e)	(+e)	(-A)	(P1)	(+C)	(-b)	(-d)	(-E)	(+f)	(+i)	(+F)	(-g)	(+B)	(+a)	(+D)	(-c)	(-h)
(+f)	(+f)	(+B)	(-C)	(P1)	(-c)	(+E)	(-d)	(-e)	(-h)	(+g)	(+F)	(+A)	(-D)	(+a)	(+b)	(-i)
(+F)	(+F)	(+a)	(+b)	(+c)	(M1)	(-i)	(+h)	(-g)	(+d)	(+e)	(+f)	(+D)	(-A)	(+B)	(-C)	(+E)
(+A)	(+A)	(-e)	(+d)	(+E)	(-i)	(M1)	(-C)	(-B)	(+b)	(-a)	(-D)	(-f)	(-F)	(+g)	(+h)	(+c)
(-B)	(-B)	(-f)	(-E)	(+d)	(+h)	(+C)	(M1)	(-A)	(+c)	(+D)	(-a)	(+e)	(-g)	(-F)	(+i)	(-b)
(+C)	(+C)	(+E)	(-f)	(+e)	(-g)	(+B)	(+A)	(M1)	(-D)	(+c)	(-b)	(-d)	(-h)	(-i)	(-F)	(+a)
(-a)	(-a)	(-F)	(+i)	(-h)	(-d)	(-b)	(-c)	(-D)	(P1)	(+A)	(-B)	(+g)	(-e)	(-f)	(-E)	(+C)
(-b)	(-b)	(-i)	(-F)	(+g)	(-e)	(+a)	(+D)	(-c)	(-A)	(P1)	(+C)	(+h)	(+d)	(+E)	(-f)	(+B)
(-c)	(-c)	(+h)	(-g)	(-F)	(-f)	(-D)	(+a)	(+b)	(+B)	(-C)	(P1)	(+i)	(-E)	(+d)	(+e)	(+A)
(+E)	(+E)	(+C)	(+B)	(+A)	(-D)	(-f)	(+e)	(-d)	(-g)	(-h)	(-i)	(M1)	(+c)	(-b)	(+a)	(-F)
(-i)	(-i)	(-b)	(+a)	(+D)	(-A)	(-F)	(+g)	(+h)	(-e)	(+d)	(+E)	(-c)	(P1)	(+C)	(+B)	(-f)
(+h)	(+h)	(-c)	(-D)	(+a)	(+B)	(-g)	(-F)	(+i)	(-f)	(-E)	(+d)	(+b)	(-C)	(P1)	(+A)	(+e)
(-g)	(-g)	(+D)	(-c)	(+b)	(-C)	(-h)	(-i)	(-F)	(+E)	(-f)	(+e)	(-a)	(-B)	(-A)	(P1)	(-d)
(-D)	(-D)	(+g)	(+h)	(+i)	(-E)	(+c)	(-b)	(+a)	(+C)	(+B)	(+A)	(+F)	(+f)	(-e)	(+d)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	-1	0]
[0	1	0	0]	[0	0	0	1]
[0	0	1	0]	[1	0	0	0]
[0	0	0	1]	[0	-1	0	0]

x				y				z				t			
[1	0	0	0]	[0	1	0	0]	[0	0	1	0]	[0	-1	0	0]
[0	-1	0	0]	[1	0	0	0]	[0	0	0	-1]	[1	0	0	0]
[0	0	-1	0]	[0	0	0	1]	[1	0	0	0]	[0	0	0	-1]
[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]	[0	0	1	0]

xy				xz				yz				xt				yt				zt			
[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	-1	0	0]	[1	0	0	0]	[0	0	0	-1]
[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]	[0	-1	0	0]	[0	0	-1	0]
[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]
[0	0	1	0]	[0	-1	0	0]	[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[-1	0	0	0]

xyz				xyt				xzt				yzt			
[0	0	0	-1]	[1	0	0	0]	[0	0	0	-1]	[0	0	-1	0]
[0	0	-1	0]	[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]
[0	1	0	0]	[0	0	-1	0]	[0	1	0	0]	[-1	0	0	0]
[1	0	0	0]	[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]

Implementation #5

	(P1)	(+g)	(+h)	(+i)	(+D)	(+A)	(-B)	(+C)	(-d)	(-e)	(-f)	(+F)	(-c)	(+b)	(-a)	(-E)	
(P1)		(P1)	(+g)	(+h)	(+i)	(+D)	(+A)	(-B)	(+C)	(-d)	(-e)	(-f)	(+F)	(-c)	(+b)	(-a)	(-E)
(+g)		(+g)	(P1)	(+A)	(-B)	(-d)	(+h)	(+i)	(+F)	(+D)	(-c)	(+b)	(+C)	(-e)	(-f)	(-E)	(-a)
(+h)		(+h)	(-A)	(P1)	(+C)	(-e)	(-g)	(-F)	(+i)	(+c)	(+D)	(-a)	(+B)	(+d)	(+E)	(-f)	(-b)
(+i)		(+i)	(+B)	(-C)	(P1)	(-f)	(+F)	(-g)	(-h)	(-b)	(+a)	(+D)	(+A)	(-E)	(+d)	(+e)	(-c)
(+D)		(+D)	(+d)	(+e)	(+f)	(M1)	(-c)	(+b)	(-a)	(+g)	(+h)	(+i)	(+E)	(-A)	(+B)	(-C)	(+F)
(+A)		(+A)	(-h)	(+g)	(+F)	(-c)	(M1)	(-C)	(-B)	(+e)	(-d)	(-E)	(-i)	(-D)	(+a)	(+b)	(+f)
(-B)		(-B)	(-i)	(-F)	(+g)	(+b)	(+C)	(M1)	(-A)	(+f)	(+E)	(-d)	(+h)	(-a)	(-D)	(+c)	(-e)
(+C)		(+C)	(+F)	(-i)	(+h)	(-a)	(+B)	(+A)	(M1)	(-E)	(+f)	(-e)	(-g)	(-b)	(-c)	(-D)	(+d)
(-d)		(-d)	(-D)	(+c)	(-b)	(-g)	(-e)	(-f)	(-E)	(P1)	(+A)	(-B)	(+a)	(-h)	(-i)	(-F)	(+C)
(-e)		(-e)	(-c)	(-D)	(+a)	(-h)	(+d)	(+E)	(-f)	(-A)	(P1)	(+C)	(+b)	(+g)	(+F)	(-i)	(+B)
(-f)		(-f)	(+b)	(-a)	(-D)	(-i)	(-E)	(+d)	(+e)	(+B)	(-C)	(P1)	(+c)	(-F)	(+g)	(+h)	(+A)
(+F)		(+F)	(+C)	(+B)	(+A)	(-E)	(-i)	(+h)	(-g)	(-a)	(-b)	(-c)	(M1)	(+f)	(-e)	(+d)	(-D)
(-c)		(-c)	(-e)	(+d)	(+E)	(-A)	(-D)	(+a)	(+b)	(-h)	(+g)	(+F)	(-f)	(P1)	(+C)	(+B)	(-i)
(+b)		(+b)	(-f)	(-E)	(+d)	(+B)	(-a)	(-D)	(+c)	(-i)	(-F)	(+g)	(+e)	(-C)	(P1)	(+A)	(+h)
(-a)		(-a)	(+E)	(-f)	(+e)	(-C)	(-b)	(-c)	(-D)	(+F)	(-i)	(+h)	(-d)	(-B)	(-A)	(P1)	(-g)
(-E)		(-E)	(+a)	(+b)	(+c)	(-F)	(+f)	(-e)	(+d)	(+C)	(+B)	(+A)	(+D)	(+i)	(-h)	(+g)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	0	1]
[0	1	0	0]	[0	0	1	0]
[0	0	1	0]	[0	-1	0	0]
[0	0	0	1]	[-1	0	0	0]

x		y		z		t									
[0	0	1	0]	[0	0	0	-1]	[-1	0	0	0]	[0	0	1	0]
[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]	[0	0	0	-1]
[1	0	0	0]	[0	1	0	0]	[0	0	1	0]	[-1	0	0	0]
[0	1	0	0]	[-1	0	0	0]	[0	0	0	1]	[0	1	0	0]

xy		xz		yz		xt		yt		zt													
[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]	[0	0	-1	0]
[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[0	1	0	0]	[-1	0	0	0]	[0	0	0	1]
[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]	[0	0	1	0]	[0	0	0	-1]	[-1	0	0	0]
[0	0	1	0]	[0	-1	0	0]	[1	0	0	0]	[0	0	0	-1]	[0	0	-1	0]	[0	1	0	0]

xyz		xyt		xzt		yzt									
[0	-1	0	0]	[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]
[1	0	0	0]	[0	0	-1	0]	[0	1	0	0]	[-1	0	0	0]
[0	0	0	-1]	[0	-1	0	0]	[0	0	-1	0]	[0	0	0	1]
[0	0	1	0]	[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]

Implementation #6

	(P1)	(+g)	(+h)	(+i)	(+E)	(+A)	(-B)	(+C)	(+a)	(+b)	(+c)	(+F)	(-f)	(+e)	(-d)	(+D)	
(P1)		(P1)	(+g)	(+h)	(+i)	(+E)	(+A)	(-B)	(+C)	(+a)	(+b)	(+c)	(+F)	(-f)	(+e)	(-d)	(+D)
(+g)		(+g)	(P1)	(+A)	(-B)	(+a)	(+h)	(+i)	(+F)	(+E)	(-f)	(+e)	(+C)	(+b)	(+c)	(+D)	(-d)
(+h)		(+h)	(-A)	(P1)	(+C)	(+b)	(-g)	(-F)	(+i)	(+f)	(+E)	(-d)	(+B)	(-a)	(-D)	(+c)	(-e)
(+i)		(+i)	(+B)	(-C)	(P1)	(+c)	(+F)	(-g)	(-h)	(-e)	(+d)	(+E)	(+A)	(+D)	(-a)	(-b)	(-f)
(+E)		(+E)	(-a)	(-b)	(-c)	(M1)	(-f)	(+e)	(-d)	(+g)	(+h)	(+i)	(-D)	(-A)	(+B)	(-C)	(+F)
(+A)		(+A)	(-h)	(+g)	(+F)	(-f)	(M1)	(-C)	(-B)	(-b)	(+a)	(+D)	(-i)	(-E)	(+d)	(+e)	(-c)
(-B)		(-B)	(-i)	(-F)	(+g)	(+e)	(+C)	(M1)	(-A)	(-c)	(-D)	(+a)	(+h)	(-d)	(-E)	(+f)	(+b)
(+C)		(+C)	(+F)	(-i)	(+h)	(-d)	(+B)	(+A)	(M1)	(+D)	(-c)	(+b)	(-g)	(-e)	(-f)	(-E)	(-a)
(+a)		(+a)	(-E)	(+f)	(-e)	(-g)	(+b)	(+c)	(+D)	(P1)	(+A)	(-B)	(+d)	(-h)	(-i)	(-F)	(+C)
(+b)		(+b)	(-f)	(-E)	(+d)	(-h)	(-a)	(-D)	(+c)	(-A)	(P1)	(+C)	(+e)	(+g)	(+F)	(-i)	(+B)
(+c)		(+c)	(+e)	(-d)	(-E)	(-i)	(+D)	(-a)	(-b)	(+B)	(-C)	(P1)	(+f)	(-F)	(+g)	(+h)	(+A)
(+F)		(+F)	(+C)	(+B)	(+A)	(+D)	(-i)	(+h)	(-g)	(-d)	(-e)	(-f)	(M1)	(-c)	(+b)	(-a)	(-E)
(-f)		(-f)	(+b)	(-a)	(-D)	(-A)	(-E)	(+d)	(+e)	(-h)	(+g)	(+F)	(+c)	(P1)	(+C)	(+B)	(-i)
(+e)		(+e)	(+c)	(+D)	(-a)	(+B)	(-d)	(-E)	(+f)	(-i)	(-F)	(+g)	(-b)	(-C)	(P1)	(+A)	(+h)
(-d)		(-d)	(-D)	(+c)	(-b)	(-C)	(-e)	(-f)	(-E)	(+F)	(-i)	(+h)	(+a)	(-B)	(-A)	(P1)	(-g)
(+D)		(+D)	(+d)	(+e)	(+f)	(-F)	(-c)	(+b)	(-a)	(+C)	(+B)	(+A)	(+E)	(+i)	(-h)	(+g)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	1	0]
[0	1	0	0]	[0	0	0	-1]
[0	0	1	0]	[-1	0	0	0]
[0	0	0	1]	[0	1	0	0]

x				y				z				t			
[0	0	1	0]	[0	0	0	-1]	[-1	0	0	0]	[0	0	0	-1]
[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]	[0	0	-1	0]
[1	0	0	0]	[0	1	0	0]	[0	0	1	0]	[0	1	0	0]
[0	1	0	0]	[-1	0	0	0]	[0	0	0	1]	[1	0	0	0]

xy				xz				yz				xt				yt				zt			
[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	1	0	0]	[-1	0	0	0]	[0	0	0	1]
[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[1	0	0	0]	[0	1	0	0]	[0	0	1	0]
[0	0	0	-1]	[-1	0	0	0]	[0	-1	0	0]	[0	0	0	-1]	[0	0	-1	0]	[0	1	0	0]
[0	0	1	0]	[0	-1	0	0]	[1	0	0	0]	[0	0	-1	0]	[0	0	0	1]	[1	0	0	0]

xyz				xyt				xzt				yzt			
[0	-1	0	0]	[0	0	-1	0]	[0	1	0	0]	[-1	0	0	0]
[1	0	0	0]	[0	0	0	1]	[1	0	0	0]	[0	1	0	0]
[0	0	0	-1]	[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]
[0	0	1	0]	[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]

Implementation #7

	(P1)	(+a)	(+d)	(+g)	(+B)	(+F)	(-E)	(+D)	(-c)	(-f)	(-i)	(+C)	(-h)	(+e)	(-b)	(-A)
(P1)	(P1)	(+a)	(+d)	(+g)	(+B)	(+F)	(-E)	(+D)	(-c)	(-f)	(-i)	(+C)	(-h)	(+e)	(-b)	(-A)
(+a)	(+a)	(P1)	(+F)	(-E)	(-c)	(+d)	(+g)	(+C)	(+B)	(-h)	(+e)	(+D)	(-f)	(-i)	(-A)	(-b)
(+d)	(+d)	(-F)	(P1)	(+D)	(-f)	(-a)	(-C)	(+g)	(+h)	(+B)	(-b)	(+E)	(+c)	(+A)	(-i)	(-e)
(+g)	(+g)	(+E)	(-D)	(P1)	(-i)	(+C)	(-a)	(-d)	(-e)	(+b)	(+B)	(+F)	(-A)	(+c)	(+f)	(-h)
(+B)	(+B)	(+c)	(+f)	(+i)	(M1)	(-h)	(+e)	(-b)	(+a)	(+d)	(+g)	(+A)	(-F)	(+E)	(-D)	(+C)
(+F)	(+F)	(-d)	(+a)	(+C)	(-h)	(M1)	(-D)	(-E)	(+f)	(-c)	(-A)	(-g)	(-B)	(+b)	(+e)	(+i)
(-E)	(-E)	(-g)	(-C)	(+a)	(+e)	(+D)	(M1)	(-F)	(+i)	(+A)	(-c)	(+d)	(-b)	(-B)	(+h)	(-f)
(+D)	(+D)	(+C)	(-g)	(+d)	(-b)	(+E)	(+F)	(M1)	(-A)	(+i)	(-f)	(-a)	(-e)	(-h)	(-B)	(+c)
(-c)	(-c)	(-B)	(+h)	(-e)	(-a)	(-f)	(-i)	(-A)	(P1)	(+F)	(-E)	(+b)	(-d)	(-g)	(-C)	(+D)
(-f)	(-f)	(-h)	(-B)	(+b)	(-d)	(+c)	(+A)	(-i)	(-F)	(P1)	(+D)	(+e)	(+a)	(+C)	(-g)	(+E)
(-i)	(-i)	(+e)	(-b)	(-B)	(-g)	(-A)	(+c)	(+f)	(+E)	(-D)	(P1)	(+h)	(-C)	(+a)	(+d)	(+F)
(+C)	(+C)	(+D)	(+E)	(+F)	(-A)	(-g)	(+d)	(-a)	(-b)	(-e)	(-h)	(M1)	(+i)	(-f)	(+c)	(-B)
(-h)	(-h)	(-f)	(+c)	(+A)	(-F)	(-B)	(+b)	(+e)	(-d)	(+a)	(+C)	(-i)	(P1)	(+D)	(+E)	(-g)
(+e)	(+e)	(-i)	(-A)	(+c)	(+E)	(-b)	(-B)	(+h)	(-g)	(-C)	(+a)	(+f)	(-D)	(P1)	(+F)	(+d)
(-b)	(-b)	(+A)	(-i)	(+f)	(-D)	(-e)	(-h)	(-B)	(+C)	(-g)	(+d)	(-c)	(-E)	(-F)	(P1)	(-a)
(-A)	(-A)	(+b)	(+e)	(+h)	(-C)	(+i)	(-f)	(+c)	(+D)	(+E)	(+F)	(+B)	(+g)	(-d)	(+a)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	-1	0	0]
[0	1	0	0]	[1	0	0	0]
[0	0	1	0]	[0	0	0	1]
[0	0	0	1]	[0	0	-1	0]

x				y				z				t			
[0	1	0	0]	[1	0	0	0]	[0	0	1	0]	[0	0	-1	0]
[1	0	0	0]	[0	-1	0	0]	[0	0	0	1]	[0	0	0	-1]
[0	0	0	-1]	[0	0	-1	0]	[1	0	0	0]	[1	0	0	0]
[0	0	-1	0]	[0	0	0	1]	[0	1	0	0]	[0	1	0	0]

xy				xz				yz				xt				yt				zt			
[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]	[0	0	0	-1]	[0	0	-1	0]	[1	0	0	0]
[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	0	-1	0]	[0	0	0	1]	[0	1	0	0]
[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]	[0	-1	0	0]	[-1	0	0	0]	[0	0	-1	0]
[0	0	1	0]	[-1	0	0	0]	[0	1	0	0]	[-1	0	0	0]	[0	1	0	0]	[0	0	0	-1]

xyz				xyt				xzt				yzt			
[0	0	0	-1]	[0	0	0	1]	[0	1	0	0]	[1	0	0	0]
[0	0	1	0]	[0	0	-1	0]	[1	0	0	0]	[0	-1	0	0]
[0	-1	0	0]	[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]
[1	0	0	0]	[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]

Implementation #8

	(P1)	(+a)	(+d)	(+g)	(+A)	(+F)	(-E)	(+D)	(+b)	(+e)	(+h)	(+C)	(-i)	(+f)	(-c)	(+B)
(P1)	(P1)	(+a)	(+d)	(+g)	(+A)	(+F)	(-E)	(+D)	(+b)	(+e)	(+h)	(+C)	(-i)	(+f)	(-c)	(+B)
(+a)	(+a)	(P1)	(+F)	(-E)	(+b)	(+d)	(+g)	(+C)	(+A)	(-i)	(+f)	(+D)	(+e)	(+h)	(+B)	(-c)
(+d)	(+d)	(-F)	(P1)	(+D)	(+e)	(-a)	(-C)	(+g)	(+i)	(+A)	(-c)	(+E)	(-b)	(-B)	(+h)	(-f)
(+g)	(+g)	(+E)	(-D)	(P1)	(+h)	(+C)	(-a)	(-d)	(-f)	(+c)	(+A)	(+F)	(+B)	(-b)	(-e)	(-i)
(+A)	(+A)	(-b)	(-e)	(-h)	(M1)	(-i)	(+f)	(-c)	(+a)	(+d)	(+g)	(-B)	(-F)	(+E)	(-D)	(+C)
(+F)	(+F)	(-d)	(+a)	(+C)	(-i)	(M1)	(-D)	(-E)	(-e)	(+b)	(+B)	(-g)	(-A)	(+c)	(+f)	(-h)
(-E)	(-E)	(-g)	(-C)	(+a)	(+f)	(+D)	(M1)	(-F)	(-h)	(-B)	(+b)	(+d)	(-c)	(-A)	(+i)	(+e)
(+D)	(+D)	(+C)	(-g)	(+d)	(-c)	(+E)	(+F)	(M1)	(+B)	(-h)	(+e)	(-a)	(-f)	(-i)	(-A)	(-b)
(+b)	(+b)	(-A)	(+i)	(-f)	(-a)	(+e)	(+h)	(+B)	(P1)	(+F)	(-E)	(+c)	(-d)	(-g)	(-C)	(+D)
(+e)	(+e)	(-i)	(-A)	(+c)	(-d)	(-b)	(-B)	(+h)	(-F)	(P1)	(+D)	(+f)	(+a)	(+C)	(-g)	(+E)
(+h)	(+h)	(+f)	(-c)	(-A)	(-g)	(+B)	(-b)	(-e)	(+E)	(-D)	(P1)	(+i)	(-C)	(+a)	(+d)	(+F)
(+C)	(+C)	(+D)	(+E)	(+F)	(+B)	(-g)	(+d)	(-a)	(-c)	(-f)	(-i)	(M1)	(-h)	(+e)	(-b)	(-A)
(-i)	(-i)	(+e)	(-b)	(-B)	(-F)	(-A)	(+c)	(+f)	(-d)	(+a)	(+C)	(+h)	(P1)	(+D)	(+E)	(-g)
(+f)	(+f)	(+h)	(+B)	(-b)	(+E)	(-c)	(-A)	(+i)	(-g)	(-C)	(+a)	(-e)	(-D)	(P1)	(+F)	(+d)
(-c)	(-c)	(-B)	(+h)	(-e)	(-D)	(-f)	(-i)	(-A)	(+C)	(-g)	(+d)	(+b)	(-E)	(-F)	(P1)	(-a)
(+B)	(+B)	(+c)	(+f)	(+i)	(-C)	(-h)	(+e)	(-b)	(+D)	(+E)	(+F)	(+A)	(+g)	(-d)	(+a)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	-1	0]
[0	1	0	0]	[0	0	0	-1]
[0	0	1	0]	[1	0	0	0]
[0	0	0	1]	[0	1	0	0]

x				y				z				t			
[0	1	0	0]	[1	0	0	0]	[0	0	1	0]	[0	1	0	0]
[1	0	0	0]	[0	-1	0	0]	[0	0	0	1]	[-1	0	0	0]
[0	0	0	-1]	[0	0	-1	0]	[1	0	0	0]	[0	0	0	-1]
[0	0	-1	0]	[0	0	0	1]	[0	1	0	0]	[0	0	1	0]

xy				xz				yz				xt				yt				zt			
[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]	[0	1	0	0]	[0	0	0	-1]
[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	1	0	0]	[1	0	0	0]	[0	0	1	0]
[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]	[0	0	-1	0]	[0	0	0	1]	[0	1	0	0]
[0	0	1	0]	[-1	0	0	0]	[0	1	0	0]	[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]

xyz				xyt				xzt				yzt			
[0	0	0	-1]	[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]
[0	0	1	0]	[0	1	0	0]	[0	0	0	-1]	[0	0	-1	0]
[0	-1	0	0]	[0	0	-1	0]	[1	0	0	0]	[0	-1	0	0]
[1	0	0	0]	[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]

Implementation #9

	(P1)	(+b)	(+e)	(+h)	(+C)	(+F)	(-E)	(+D)	(+c)	(+f)	(+i)	(+B)	(-g)	(+d)	(-a)	(+A)
(P1)	(P1)	(+b)	(+e)	(+h)	(+C)	(+F)	(-E)	(+D)	(+c)	(+f)	(+i)	(+B)	(-g)	(+d)	(-a)	(+A)
(+b)	(+b)	(P1)	(+F)	(-E)	(+c)	(+e)	(+h)	(+B)	(+C)	(-g)	(+d)	(+D)	(+f)	(+i)	(+A)	(-a)
(+e)	(+e)	(-F)	(P1)	(+D)	(+f)	(-b)	(-B)	(+h)	(+g)	(+C)	(-a)	(+E)	(-c)	(-A)	(+i)	(-d)
(+h)	(+h)	(+E)	(-D)	(P1)	(+i)	(+B)	(-b)	(-e)	(-d)	(+a)	(+C)	(+F)	(+A)	(-c)	(-f)	(-g)
(+C)	(+C)	(-c)	(-f)	(-i)	(M1)	(-g)	(+d)	(-a)	(+b)	(+e)	(+h)	(-A)	(-F)	(+E)	(-D)	(+B)
(+F)	(+F)	(-e)	(+b)	(+B)	(-g)	(M1)	(-D)	(-E)	(-f)	(+c)	(+A)	(-h)	(-C)	(+a)	(+d)	(-i)
(-E)	(-E)	(-h)	(-B)	(+b)	(+d)	(+D)	(M1)	(-F)	(-i)	(-A)	(+c)	(+e)	(-a)	(-C)	(+g)	(+f)
(+D)	(+D)	(+B)	(-h)	(+e)	(-a)	(+E)	(+F)	(M1)	(+A)	(-i)	(+f)	(-b)	(-d)	(-g)	(-C)	(-c)
(+c)	(+c)	(-C)	(+g)	(-d)	(-b)	(+f)	(+i)	(+A)	(P1)	(+F)	(-E)	(+a)	(-e)	(-h)	(-B)	(+D)
(+f)	(+f)	(-g)	(-C)	(+a)	(-e)	(-c)	(-A)	(+i)	(-F)	(P1)	(+D)	(+d)	(+b)	(+B)	(-h)	(+E)
(+i)	(+i)	(+d)	(-a)	(-C)	(-h)	(+A)	(-c)	(-f)	(+E)	(-D)	(P1)	(+g)	(-B)	(+b)	(+e)	(+F)
(+B)	(+B)	(+D)	(+E)	(+F)	(+A)	(-h)	(+e)	(-b)	(-a)	(-d)	(-g)	(M1)	(-i)	(+f)	(-c)	(-C)
(-g)	(-g)	(+f)	(-c)	(-A)	(-F)	(-C)	(+a)	(+d)	(-e)	(+b)	(+B)	(+i)	(P1)	(+D)	(+E)	(-h)
(+d)	(+d)	(+i)	(+A)	(-c)	(+E)	(-a)	(-C)	(+g)	(-h)	(-B)	(+b)	(-f)	(-D)	(P1)	(+F)	(+e)
(-a)	(-a)	(-A)	(+i)	(-f)	(-D)	(-d)	(-g)	(-C)	(+B)	(-h)	(+e)	(+c)	(-E)	(-F)	(P1)	(-b)
(+A)	(+A)	(+a)	(+d)	(+g)	(-B)	(-i)	(+f)	(-c)	(+D)	(+E)	(+F)	(+C)	(+h)	(-e)	(+b)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	1	0	0]
[0	1	0	0]	[-1	0	0	0]
[0	0	1	0]	[0	0	0	-1]
[0	0	0	1]	[0	0	1	0]

x				y				z				t			
[-1	0	0	0]	[0	1	0	0]	[0	0	0	-1]	[0	0	0	-1]
[0	1	0	0]	[1	0	0	0]	[0	0	1	0]	[0	0	1	0]
[0	0	-1	0]	[0	0	0	1]	[0	1	0	0]	[0	-1	0	0]
[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]	[1	0	0	0]

xy				xz				yz				xt				yt				zt			
[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]	[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]
[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	0	1	0]	[0	0	0	-1]	[0	-1	0	0]
[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]	[0	1	0	0]	[1	0	0	0]	[0	0	1	0]
[0	0	1	0]	[-1	0	0	0]	[0	1	0	0]	[1	0	0	0]	[0	-1	0	0]	[0	0	0	1]

xyz				xyt				xzt				yzt			
[0	0	-1	0]	[0	0	-1	0]	[1	0	0	0]	[0	-1	0	0]
[0	0	0	-1]	[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]
[1	0	0	0]	[-1	0	0	0]	[0	0	-1	0]	[0	0	0	1]
[0	1	0	0]	[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]

Implementation #10

	(P1)	(+b)	(+e)	(+h)	(+A)	(+F)	(-E)	(+D)	(-a)	(-d)	(-g)	(+B)	(-i)	(+f)	(-c)	(-C)
(P1)	(P1)	(+b)	(+e)	(+h)	(+A)	(+F)	(-E)	(+D)	(-a)	(-d)	(-g)	(+B)	(-i)	(+f)	(-c)	(-C)
(+b)	(+b)	(P1)	(+F)	(-E)	(-a)	(+e)	(+h)	(+B)	(+A)	(-i)	(+f)	(+D)	(-d)	(-g)	(-C)	(-c)
(+e)	(+e)	(-F)	(P1)	(+D)	(-d)	(-b)	(-B)	(+h)	(+i)	(+A)	(-c)	(+E)	(+a)	(+C)	(-g)	(-f)
(+h)	(+h)	(+E)	(-D)	(P1)	(-g)	(+B)	(-b)	(-e)	(-f)	(+c)	(+A)	(+F)	(-C)	(+a)	(+d)	(-i)
(+A)	(+A)	(+a)	(+d)	(+g)	(M1)	(-i)	(+f)	(-c)	(+b)	(+e)	(+h)	(+C)	(-F)	(+E)	(-D)	(+B)
(+F)	(+F)	(-e)	(+b)	(+B)	(-i)	(M1)	(-D)	(-E)	(+d)	(-a)	(-C)	(-h)	(-A)	(+c)	(+f)	(+g)
(-E)	(-E)	(-h)	(-B)	(+b)	(+f)	(+D)	(M1)	(-F)	(+g)	(+C)	(-a)	(+e)	(-c)	(-A)	(+i)	(-d)
(+D)	(+D)	(+B)	(-h)	(+e)	(-c)	(+E)	(+F)	(M1)	(-C)	(+g)	(-d)	(-b)	(-f)	(-i)	(-A)	(+a)
(-a)	(-a)	(-A)	(+i)	(-f)	(-b)	(-d)	(-g)	(-C)	(P1)	(+F)	(-E)	(+c)	(-e)	(-h)	(-B)	(+D)
(-d)	(-d)	(-i)	(-A)	(+c)	(-e)	(+a)	(+C)	(-g)	(-F)	(P1)	(+D)	(+f)	(+b)	(+B)	(-h)	(+E)
(-g)	(-g)	(+f)	(-c)	(-A)	(-h)	(-C)	(+a)	(+d)	(+E)	(-D)	(P1)	(+i)	(-B)	(+b)	(+e)	(+F)
(+B)	(+B)	(+D)	(+E)	(+F)	(-C)	(-h)	(+e)	(-b)	(-c)	(-f)	(-i)	(M1)	(+g)	(-d)	(+a)	(-A)
(-i)	(-i)	(-d)	(+a)	(+C)	(-F)	(-A)	(+c)	(+f)	(-e)	(+b)	(+B)	(-g)	(P1)	(+D)	(+E)	(-h)
(+f)	(+f)	(-g)	(-C)	(+a)	(+E)	(-c)	(-A)	(+i)	(-h)	(-B)	(+b)	(+d)	(-D)	(P1)	(+F)	(+e)
(-c)	(-c)	(+C)	(-g)	(+d)	(-D)	(-f)	(-i)	(-A)	(+B)	(-h)	(+e)	(-a)	(-E)	(-F)	(P1)	(-b)
(-C)	(-C)	(+c)	(+f)	(+i)	(-B)	(+g)	(-d)	(+a)	(+D)	(+E)	(+F)	(+A)	(+h)	(-e)	(+b)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	0	1]
[0	1	0	0]	[0	0	-1	0]
[0	0	1	0]	[0	1	0	0]
[0	0	0	1]	[-1	0	0	0]

x				y				z				t			
[-1	0	0	0]	[0	1	0	0]	[0	0	0	-1]	[0	1	0	0]
[0	1	0	0]	[1	0	0	0]	[0	0	1	0]	[-1	0	0	0]
[0	0	-1	0]	[0	0	0	1]	[0	1	0	0]	[0	0	0	-1]
[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]	[0	0	1	0]

xy				xz				yz				xt				yt				zt			
[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]	[0	-1	0	0]	[-1	0	0	0]	[0	0	-1	0]
[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[-1	0	0	0]	[0	1	0	0]	[0	0	0	-1]
[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]	[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]
[0	0	1	0]	[-1	0	0	0]	[0	1	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	-1	0	0]

xyz				xyt				xzt				yzt			
[0	0	-1	0]	[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]
[0	0	0	-1]	[0	1	0	0]	[0	0	0	-1]	[0	0	-1	0]
[1	0	0	0]	[0	0	-1	0]	[1	0	0	0]	[0	-1	0	0]
[0	1	0	0]	[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]

Implementation #11

	(P1)	(+c)	(+f)	(+i)	(+B)	(+F)	(-E)	(+D)	(+a)	(+d)	(+g)	(+A)	(-h)	(+e)	(-b)	(+C)
(P1)	(P1)	(+c)	(+f)	(+i)	(+B)	(+F)	(-E)	(+D)	(+a)	(+d)	(+g)	(+A)	(-h)	(+e)	(-b)	(+C)
(+c)	(+c)	(P1)	(+F)	(-E)	(+a)	(+f)	(+i)	(+A)	(+B)	(-h)	(+e)	(+D)	(+d)	(+g)	(+C)	(-b)
(+f)	(+f)	(-F)	(P1)	(+D)	(+d)	(-c)	(-A)	(+i)	(+h)	(+B)	(-b)	(+E)	(-a)	(-C)	(+g)	(-e)
(+i)	(+i)	(+E)	(-D)	(P1)	(+g)	(+A)	(-c)	(-f)	(-e)	(+b)	(+B)	(+F)	(+C)	(-a)	(-d)	(-h)
(+B)	(+B)	(-a)	(-d)	(-g)	(M1)	(-h)	(+e)	(-b)	(+c)	(+f)	(+i)	(-C)	(-F)	(+E)	(-D)	(+A)
(+F)	(+F)	(-f)	(+c)	(+A)	(-h)	(M1)	(-D)	(-E)	(-d)	(+a)	(+C)	(-i)	(-B)	(+b)	(+e)	(-g)
(-E)	(-E)	(-i)	(-A)	(+c)	(+e)	(+D)	(M1)	(-F)	(-g)	(-C)	(+a)	(+f)	(-b)	(-B)	(+h)	(+d)
(+D)	(+D)	(+A)	(-i)	(+f)	(-b)	(+E)	(+F)	(M1)	(+C)	(-g)	(+d)	(-c)	(-e)	(-h)	(-B)	(-a)
(+a)	(+a)	(-B)	(+h)	(-e)	(-c)	(+d)	(+g)	(+C)	(P1)	(+F)	(-E)	(+b)	(-f)	(-i)	(-A)	(+D)
(+d)	(+d)	(-h)	(-B)	(+b)	(-f)	(-a)	(-C)	(+g)	(-F)	(P1)	(+D)	(+e)	(+c)	(+A)	(-i)	(+E)
(+g)	(+g)	(+e)	(-b)	(-B)	(-i)	(+C)	(-a)	(-d)	(+E)	(-D)	(P1)	(+h)	(-A)	(+c)	(+f)	(+F)
(+A)	(+A)	(+D)	(+E)	(+F)	(+C)	(-i)	(+f)	(-c)	(-b)	(-e)	(-h)	(M1)	(-g)	(+d)	(-a)	(-B)
(-h)	(-h)	(+d)	(-a)	(-C)	(-F)	(-B)	(+b)	(+e)	(-f)	(+c)	(+A)	(+g)	(P1)	(+D)	(+E)	(-i)
(+e)	(+e)	(+g)	(+C)	(-a)	(+E)	(-b)	(-B)	(+h)	(-i)	(-A)	(+c)	(-d)	(-D)	(P1)	(+F)	(+f)
(-b)	(-b)	(-C)	(+g)	(-d)	(-D)	(-e)	(-h)	(-B)	(+A)	(-i)	(+f)	(+a)	(-E)	(-F)	(P1)	(-c)
(+C)	(+C)	(+b)	(+e)	(+h)	(-A)	(-g)	(+d)	(-a)	(+D)	(+E)	(+F)	(+B)	(+i)	(-f)	(+c)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	0	-1]
[0	1	0	0]	[0	0	1	0]
[0	0	1	0]	[0	-1	0	0]
[0	0	0	1]	[1	0	0	0]

x		y		z		t									
[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]	[0	0	-1	0]
[0	0	1	0]	[0	0	0	-1]	[0	-1	0	0]	[0	0	0	-1]
[0	1	0	0]	[1	0	0	0]	[0	0	1	0]	[1	0	0	0]
[1	0	0	0]	[0	-1	0	0]	[0	0	0	1]	[0	1	0	0]

xy			xz			yz			xt			yt			zt								
[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]	[0	1	0	0]	[1	0	0	0]	[0	0	1	0]
[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[1	0	0	0]	[0	-1	0	0]	[0	0	0	1]
[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]	[0	0	0	-1]	[0	0	-1	0]	[1	0	0	0]
[0	0	1	0]	[-1	0	0	0]	[0	1	0	0]	[0	0	-1	0]	[0	0	0	1]	[0	1	0	0]

xyz				xyt				xzt				yzt			
[0	1	0	0]	[0	0	0	1]	[0	1	0	0]	[1	0	0	0]
[-1	0	0	0]	[0	0	-1	0]	[1	0	0	0]	[0	-1	0	0]
[0	0	0	-1]	[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]
[0	0	1	0]	[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]

Implementation #12

	(P1)	(+c)	(+f)	(+i)	(+C)	(+F)	(-E)	(+D)	(-b)	(-e)	(-h)	(+A)	(-g)	(+d)	(-a)	(-B)	
(P1)		(P1)	(+c)	(+f)	(+i)	(+C)	(+F)	(-E)	(+D)	(-b)	(-e)	(-h)	(+A)	(-g)	(+d)	(-a)	(-B)
(+c)		(+c)	(P1)	(+F)	(-E)	(-b)	(+f)	(+i)	(+A)	(+C)	(-g)	(+d)	(+D)	(-e)	(-h)	(-B)	(-a)
(+f)		(+f)	(-F)	(P1)	(+D)	(-e)	(-c)	(-A)	(+i)	(+g)	(+C)	(-a)	(+E)	(+b)	(+B)	(-h)	(-d)
(+i)		(+i)	(+E)	(-D)	(P1)	(-h)	(+A)	(-c)	(-f)	(-d)	(+a)	(+C)	(+F)	(-B)	(+b)	(+e)	(-g)
(+C)		(+C)	(+b)	(+e)	(+h)	(M1)	(-g)	(+d)	(-a)	(+c)	(+f)	(+i)	(+B)	(-F)	(+E)	(-D)	(+A)
(+F)		(+F)	(-f)	(+c)	(+A)	(-g)	(M1)	(-D)	(-E)	(+e)	(-b)	(-B)	(-i)	(-C)	(+a)	(+d)	(+h)
(-E)		(-E)	(-i)	(-A)	(+c)	(+d)	(+D)	(M1)	(-F)	(+h)	(+B)	(-b)	(+f)	(-a)	(-C)	(+g)	(-e)
(+D)		(+D)	(+A)	(-i)	(+f)	(-a)	(+E)	(+F)	(M1)	(-B)	(+h)	(-e)	(-c)	(-d)	(-g)	(-C)	(+b)
(-b)		(-b)	(-C)	(+g)	(-d)	(-c)	(-e)	(-h)	(-B)	(P1)	(+F)	(-E)	(+a)	(-f)	(-i)	(-A)	(+D)
(-e)		(-e)	(-g)	(-C)	(+a)	(-f)	(+b)	(+B)	(-h)	(-F)	(P1)	(+D)	(+d)	(+c)	(+A)	(-i)	(+E)
(-h)		(-h)	(+d)	(-a)	(-C)	(-i)	(-B)	(+b)	(+e)	(+E)	(-D)	(P1)	(+g)	(-A)	(+c)	(+f)	(+F)
(+A)		(+A)	(+D)	(+E)	(+F)	(-B)	(-i)	(+f)	(-c)	(-a)	(-d)	(-g)	(M1)	(+h)	(-e)	(+b)	(-C)
(-g)		(-g)	(-e)	(+b)	(+B)	(-F)	(-C)	(+a)	(+d)	(-f)	(+c)	(+A)	(-h)	(P1)	(+D)	(+E)	(-i)
(+d)		(+d)	(-h)	(-B)	(+b)	(+E)	(-a)	(-C)	(+g)	(-i)	(-A)	(+c)	(+e)	(-D)	(P1)	(+F)	(+f)
(-a)		(-a)	(+B)	(-h)	(+e)	(-D)	(-d)	(-g)	(-C)	(+A)	(-i)	(+f)	(-b)	(-E)	(-F)	(P1)	(-c)
(-B)		(-B)	(+a)	(+d)	(+g)	(-A)	(+h)	(-e)	(+b)	(+D)	(+E)	(+F)	(+C)	(+i)	(-f)	(+c)	(M1)

Unity				xyzt			
[1	0	0	0]	[0	0	1	0]
[0	1	0	0]	[0	0	0	1]
[0	0	1	0]	[-1	0	0	0]
[0	0	0	1]	[0	-1	0	0]

x		y		z		t									
[0	0	0	1]	[0	0	1	0]	[-1	0	0	0]	[0	0	0	-1]
[0	0	1	0]	[0	0	0	-1]	[0	-1	0	0]	[0	0	1	0]
[0	1	0	0]	[1	0	0	0]	[0	0	1	0]	[0	-1	0	0]
[1	0	0	0]	[0	-1	0	0]	[0	0	0	1]	[1	0	0	0]

xy			xz			yz			xt			yt			zt								
[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]	[1	0	0	0]	[0	-1	0	0]	[0	0	0	1]
[1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]	[0	0	-1	0]
[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]	[0	0	1	0]	[0	0	0	-1]	[0	-1	0	0]
[0	0	1	0]	[-1	0	0	0]	[0	1	0	0]	[0	0	0	-1]	[0	0	-1	0]	[1	0	0	0]

xyz				xyt				xzt				yzt			
[0	1	0	0]	[0	0	-1	0]	[1	0	0	0]	[0	-1	0	0]
[-1	0	0	0]	[0	0	0	-1]	[0	-1	0	0]	[-1	0	0	0]
[0	0	0	-1]	[-1	0	0	0]	[0	0	-1	0]	[0	0	0	1]
[0	0	1	0]	[0	-1	0	0]	[0	0	0	1]	[0	0	1	0]

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