Product Preserving Hyperbolic Transform

Kurt Nalty

July 28, 2013

Right Hyperbola

Hyperbolic transforms preserve the product of two items.

One common hyperbola is defined as

$$xy = a^2$$

where the product to be conserved is a^2, with the point of closest approach to the origin being $r_{min} = a\sqrt{2}$.

Using a parametric description, we can define the same hyperbola by

$$\begin{aligned}
r^2 \sin(2\theta) &= 2a^2 \\
r &= \frac{a\sqrt{2}}{\sqrt{\sin(2\theta)}} \\
r &= \frac{a}{\sqrt{\sin(\theta)\cos(\theta)}} \\
x &= r\cos(\theta) = a\sqrt{\cot(\theta)} \\
y &= r\sin(\theta) = a\sqrt{\tan(\theta)}
\end{aligned}$$

The point of closest approach to the origin is at $\theta = 45$ degrees. Angle 0 corresponds to the lower right of the branch, while 90 degrees corresponds to the upper left.

Figure 1 shows two such parabolas.

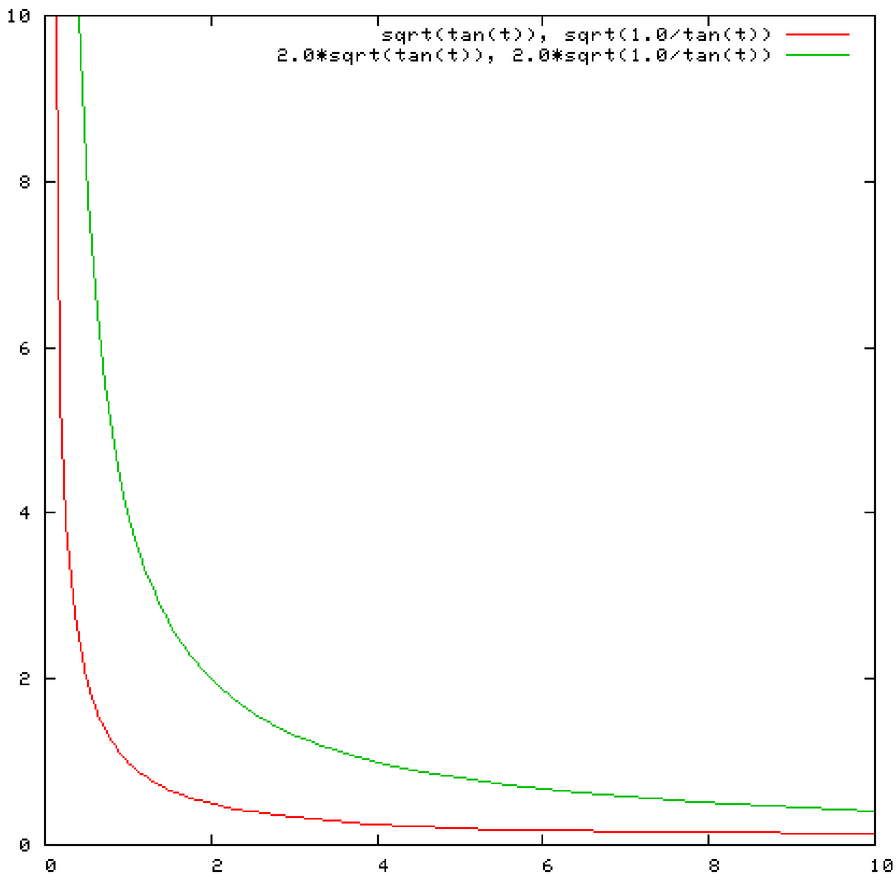

Figure 1: Simple Quadrant I Hyperbola

Illustration of Conserved Product Mixing

In conventional electromagnetism, we have a relationship between the speed of light, the electric permittivity and the magnetic permeability.

$$\epsilon\mu = \frac{1}{c^2}$$

I wish to explore systems where the values of ϵ and μ differ from local values, yet maintain constant light speed. My generating function for these new paramters is given by

$$\begin{aligned} \epsilon' &= \epsilon\sqrt{\cot(\theta)} \\ \mu' &= \mu\sqrt{\tan(\theta)} \end{aligned}$$

The product remains constant. The mixing parameter is θ.